

## Portfolio Important formulas

### MPT

1. Covariance between two stocks

$$\text{Cov}(x, y) = \text{net risk of stock } x \times \text{net risk of stock } y$$

$$2. \quad \rho_{xy} = \frac{\text{Cov}(x, y)}{\sigma_x \cdot \sigma_y}$$

$$3. \quad \text{Cov}(x, y) = \rho_{xy} \cdot \sigma_x \cdot \sigma_y$$

4. Weights of minimum risk portfolio

$$w_x = \frac{\text{Risk reduction of } y}{\text{Risk reduction of } x + \text{Risk reduction of } y} = \frac{\sigma_y^2 - \text{Cov}(x, y)}{\sigma_x^2 - \text{Cov}(x, y) + \sigma_y^2 - \text{Cov}(x, y)}$$

$$w_y = 1 - w_x$$

5. Return of portfolio =  $w_x \cdot R_x + w_y \cdot R_y + \dots$

$$6. \quad \text{S.D. of portfolio (2 stocks)} = \sqrt{\tilde{w}_x^2 \tilde{\sigma}_x^2 + \tilde{w}_y^2 \tilde{\sigma}_y^2 + 2\tilde{w}_x \tilde{w}_y \text{Cov}(x, y)}$$

$$7. \quad \text{S.D. of portfolio (3 stocks)} = \sqrt{\tilde{w}_x^2 \tilde{\sigma}_x^2 + \tilde{w}_y^2 \tilde{\sigma}_y^2 + \tilde{w}_z^2 \tilde{\sigma}_z^2 + 2\tilde{w}_x \tilde{w}_y \text{Cov}(x, y) + 2\tilde{w}_y \tilde{w}_z \text{Cov}(y, z) + 2\tilde{w}_x \tilde{w}_z \text{Cov}(x, z)}$$

## SEM and CAPM

$$\text{Beta} = \frac{\text{Changes in stock return}}{\text{Changes in market return}} ; \sigma_m$$

$$\text{Beta} = \frac{\text{Net risk of stock}}{\text{S.D. of market}}$$

↓

$$\text{Net risk of stock} = \beta_x \cdot \sigma_m$$

$$\text{Beta} = \frac{\text{Cov}(x, m)}{\sigma_m^2}$$

$$\text{Cov}(x, m) = \text{Beta} \times \sigma_m^2$$

$$\begin{aligned} \text{Cov}(x, y) &= \text{Net risk of stock } x \times \text{Net risk of stock } y \\ &= \beta_x \cdot \sigma_m \times \beta_y \cdot \sigma_m \\ &= \beta_x \cdot \beta_y \cdot \sigma_m^2 \end{aligned}$$

$$M_{xm} = \frac{\text{Cov}(x, m)}{\beta_x \cdot \sigma_m}$$

$$M_{xm} = \frac{\text{net risk of stock } x \times \text{net risk of market } m}{\beta_x \cdot \sigma_m}$$

$$\therefore \text{net risk of stock } x = M_{xm} \cdot \sigma_m$$

$$\text{net risk of stock} = \beta_x \cdot \sigma_m = M_{xm} \cdot \sigma_m$$

$$\therefore \beta_x = \frac{M_{xm} \cdot \sigma_m}{\sigma_m}$$

## Characteristic line

$$\alpha = \bar{R}_i - \beta \cdot R_m$$

The equation of characteristic line:  $\bar{R}_i = \alpha + \beta \cdot R_m$

Don't bill  $R_m$

$$\begin{aligned} \text{Total Risk of a stock} &= \text{Systematic Risk of stock} + \text{Unsystematic Risk of stock} \\ \downarrow \\ \sigma_a^2 &= \beta^2 \cdot \sigma_m^2 + \sigma_{e_a}^2 \end{aligned}$$

$$\text{Unsystematic Risk of stock} = \text{Total Risk of stock} - \text{Syst. Risk of stock}$$

## Portfolio

$$\text{Systematic Risk of Portfolio} = \beta_p^2 \cdot \sigma_m^2$$

$$\text{Unsystematic Risk of Portfolio} = w_A^2 \cdot \sigma_{e_A}^2 + w_B^2 \cdot \sigma_{e_B}^2 + \dots$$

## CAPM

The equation of Security market line:  $E(R) = R_f + (R_m - R_f) \beta$

Don't bill Beta

$$\text{Sharpe Ratio (SR)} = \frac{\bar{R}_i - R_f}{\text{S.D.}}$$

$$\text{Treynor Ratio (TR)} = \frac{\bar{R}_i - R_f}{\text{Beta}}$$

$$\text{Jensen's Alpha} : \alpha = \bar{R}_i - R_f + (R_m - R_f) \beta$$

$$M_{xy} = M_{xm} \times M_{ym}$$

$$\text{Co-efficient of determination} = r^2$$

$$\text{Co-efficient of variation} = \frac{SD}{\text{mean}} \times 100$$

### APT

$$E(R) = R_f + \beta_1 \lambda_1 + \beta_2 \lambda_2 + \beta_3 \lambda_3 \dots$$

OR

$$R_f + \left\{ \text{Factor 1 Risk Premium} \times \text{Factor 1 sensitivity} \right\} + \\ \left\{ \text{Factor 2 Risk Premium} \times \text{Factor 2 sensitivity} \right\} + \\ \left\{ \text{Factor 3 Risk Premium} \times \text{Factor 3 sensitivity} \right\} \\ \dots$$

### Capital market line

$$\text{Equation is } E(R) = R_f + (R_m - R_f) \frac{\sigma_P}{\sigma_m}$$

Inv in  $R_f$  and Inv in  $R_m \rightarrow$  Lending Portfolio (under market portfolio)

Borrow in  $R_f$  and Inv in  $R_m \rightarrow$  Borrowing Portfolio (over market portfolio)

# Efficient Frontier Line

<u>Combinations</u>	<u>Returns</u>	<u>Risk</u>
A	8%	5%
<del>B</del>	<del>8%</del>	<del>6%</del>
<del>C</del>	<del>7%</del>	<del>6%</del>
D	10%	8%
<del>E</del>	<del>9%</del>	<del>8%</del>
F	12%	10%
<del>G</del>	<del>12%</del>	<del>12%</del>
H	14%	14%

Returns

